2018-2019 In-Class Test Solutions

1.

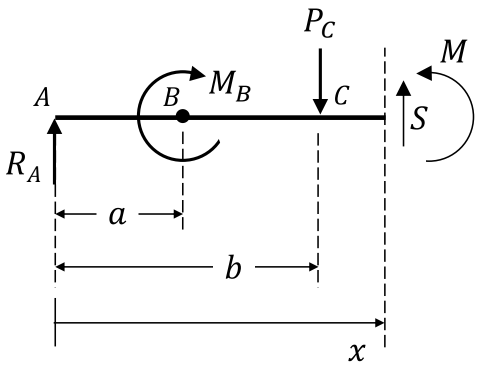
A. elastic-perfectly-plastic

2.

C. $EI\frac{d^2y}{dx^2} = R_A x + M_B\langle x-a\rangle^0 - P_C\langle x-b\rangle$

Solution Q2

Sectioning the beam after the last discontinuity (taking origin at left hand side of the beam) and drawing a Free Body Diagram of left hand side of section:



Taking moments about the section position (and applying Macauley's convention):

$$M + P_C\langle x-b\rangle = M_B\langle x-a\rangle^0 + R_A x$$

$$\therefore M = R_A x + M_B\langle x-a\rangle^0 - P_C\langle x-b\rangle$$

Substituting this into the deflection of beams equation:

$$\boldsymbol{EI\frac{d^2y}{dx^2} = R_A x + M_B\langle x-a\rangle^0 - P_C\langle x-b\rangle}$$

3.

D. False

Solution Q3

S_{range} and S_{amp} are labelled on the diagram the wrong way around.

4.

D. 162 MPa

Solution Q4

Given $\sigma_z = 150$ MPa and $\tau_{z\theta} = 45$ MPa

$$C = \frac{\sigma_x + \sigma_y}{2} = \frac{150 + 0}{2} = 75 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{z\theta}^2} = \sqrt{75^2 + 45^2} = 87.46 \text{ MPa}$$

$$\sigma_1 = C + R = 175 + 87.46 = 162.46 \approx \mathbf{162\ MPa}$$

5.

D. 0.8×10^{-3} m

Solution Q5

$$\delta = L\alpha\Delta T = 1 \times 16 \times 10^{-6} \times 50 = \mathbf{8 \times 10^{-4}\ m}$$

6.

D. $y = \frac{1}{EI}\left(R_A\frac{x^3}{6} + P\frac{\langle x-4\rangle^3}{6} - w\frac{\langle x-8\rangle^4}{24} + Ax + B\right)$

Solution Q6

$$EI\frac{d^2y}{dx^2} = R_A x + P\langle x-4\rangle - w\frac{\langle x-8\rangle^2}{2}$$

Integrating this gives:

$$EI\frac{dy}{dx} = R_A\frac{x^2}{2} + P\frac{\langle x-4\rangle^2}{2} - w\frac{\langle x-8\rangle^3}{6} + A$$

Integrating again gives:

$$EIy = R_A\frac{x^3}{6} + P\frac{\langle x-4\rangle^3}{6} - w\frac{\langle x-8\rangle^4}{24} + Ax + B$$

Rearranging this gives:

$$\boldsymbol{y = \frac{1}{EI}\left(R_A\frac{x^3}{6} + P\frac{\langle x-4\rangle^3}{6} - w\frac{\langle x-8\rangle^4}{24} + Ax + B\right)}$$

7.

C. AB

8.

B. 50 MPa

Solution Q8

Torsional shear stress

$$\tau = \frac{Tr}{J} = \frac{32 \times 500 \times 20 \times \ 10^{-3}}{\pi \times (40 \times 10^{-3})^4} = 39.8 \text{ MPa}$$

Axial stress

$$\sigma_a = \frac{F}{A} = \frac{75000}{\pi \times (20 \times 10^{-3})^2} = 59.7 \text{ MPa}$$

Maximum shear stress is given by

$$\tau_{max} = R = \sqrt{\left(\frac{\sigma_a}{2}\right)^2 + \tau^2} = \sqrt{29.85^2 + 39.8^2} = 49.75 \approx \mathbf{50\ MPa}$$

9.

B. 5×10^{-3} m

Solution Q9

$$\sigma_{y(s.f)} = \frac{\sigma_y}{2} = 100 \text{ MPa}$$

For a pressure vessel under internal pressure only

$$\sigma_1 = \sigma_\theta$$

$$\sigma_2 = \sigma_a$$

$$\sigma_3 = 0$$

Tresca yield criteria states yield will occur if

$$\sigma_1 - \sigma_3 = \sigma_\theta \geq \sigma_{y(s.f)}$$

and

$$\sigma_\theta = \frac{pr}{t}$$

Therefore, to avoid yield (including s.f.)

$$t = \frac{pr}{\sigma_{y(s.f)}} = \frac{2 \times 10^6 \times 0.25}{100 \times 10^6} = \mathbf{5 \times 10^{-3} \ m}$$

10.

D. No

Solution Q10

$$K_{max} = 1.25\sigma\sqrt{\pi a}$$

Therefore,

$$K_{I_c} = 1.25\sigma\sqrt{\pi a_c}$$

(c = critical)

Rearranging,

$$a_c = \frac{\left(\frac{K_{I_c}}{1.25\sigma}\right)^2}{\pi}$$

$$= \frac{\left(\frac{55}{1.25 \times 750}\right)^2}{\pi}$$

$$= 0.001096m = 1.096mm$$

Since the crack does not exceed the critical crack length, i.e. $0.75mm < 1.096mm$, the component **will not fracture** under this load.

11.

B. is detrimental to fatigue life

12.

D. 90.45mm

Solution Q12

Moment Equilibrium:

$$M = \int_A y\sigma dA = \int_{-d/2}^{d/2} y\sigma b dy = 2\int_0^{d/2} y\sigma b dy$$

$$= 2\left\{\int_0^a y\left(\frac{\sigma_y}{a}y\right) bdy + \int_a^{d/2} y(\sigma_y)bdy\right\} = 2\sigma_y b\left\{\int_0^a \frac{y^2}{a}dy + \int_a^{d/2} ydy\right\}$$

$$= 2\sigma_y b\left\{\left[\frac{y^3}{3a}\right]_0^a + \left[\frac{y^2}{2}\right]_a^{d/2}\right\} = 2\sigma_y b\left\{\frac{d^2}{8} - \frac{a^2}{6}\right\}$$

$$\therefore 300 \times 10^6 = 2 \times 275 \times 150 \times \left\{\frac{200^2}{8} - \frac{a^2}{6}\right\}$$

$$\therefore \boldsymbol{a = 90.45mm}$$

13.

B. 45 mm

Solution Q13

von Mises yield criteria for a plane stress case

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 \geq \sigma_y^2$$

For a shaft under pure torque, the Mohr's circle is centred on the origin and σ_1 and σ_2 will be the same magnitude, *k*, and also be the maximum allowable shear stress, therefore

$$3k^2 = \sigma_y^2$$

And the value of *k* will therefore be

$$k = \frac{\sigma_y}{\sqrt{3}} = \frac{250 \times 10^6}{\sqrt{3}} = 144 \times 10^6 \text{ Pa}$$

Therefore

$$\tau = 144 \times 10^6 = \frac{Tr}{J} = \frac{2T}{\pi r^3}$$

Rearranging

$$r = \sqrt[3]{\frac{2 \times 20000}{144 \times 10^6 \pi}} = 0.0445 \text{ m} \approx \mathbf{45\ mm}$$

14.

C. – 63 MPa

Solution Q14

$$\delta_{total} = 0 = \delta_{therm} + \delta_{mech} = L\alpha\Delta T + \frac{FL}{AE}$$

Therefore

$$-\alpha\Delta T = \frac{F}{AE}$$

But $\sigma = \frac{F}{A}$, so

$$-\alpha\Delta TE = \sigma = -12 \times 10^{-6} \times 25 \times 210 \times 10^{9} = -63 \times 10^{6}\ \text{Pa} = \mathbf{-63\ MPa}$$

15.

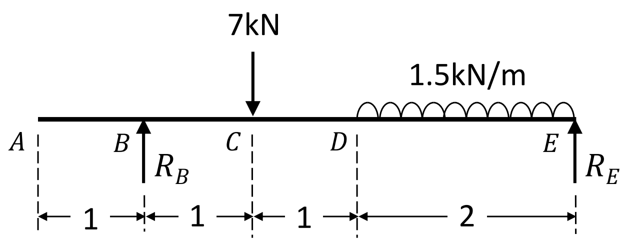
B. y-planes

16.

A. 6 kN

Solution Q16

Reaction forces will be present at positions B and E, namely R_B and R_E as shown below.



Taking moments about position E:

$$R_B \times 4 = 7 \times 3 + 1.5 \times 2 \times 1$$

$$\therefore R_B = \boldsymbol{6kN}$$

17.

A. True

18.

A. More conservative

19.

A. 68 MPa

Solution Q19

$$\delta_{total} = \delta_{initial} = \frac{F_{initial}L}{AE} = \delta_{therm} + \delta_{mech} = L\alpha\Delta T + \frac{FL}{AE}$$

Therefore

$$\frac{F_{initial}}{A} - \alpha\Delta TE = \frac{F}{A} = \frac{7500}{75 \times 10^{-6}} - 23 \times 10^{-6} \times 20 \times 70 \times 10^{9} = 67.8 \times 10^{6}\ \text{Pa} = \mathbf{68\ MPa}$$

20.

A. Yes

Solution Q20

Behaviour is assumed to be all elastic and therefore:

$$\frac{M_y}{I} = \frac{\sigma_y}{y}$$

where M_y is the moment required to cause yielding.

First yield will occur at $y = \pm\frac{d}{2}$, i.e. at the top and bottom edges:

$$\therefore M_y = \frac{\sigma_y \times I}{y} = \frac{\sigma_y \times \left(\frac{bd^3}{12}\right)}{\frac{d}{2}} = \frac{250 \times \left(\frac{150 \times 200^3}{12}\right)}{\frac{200}{2}} = 250{,}000{,}000Nmm = 250kNm$$

Since $M > M_y$, **yielding does occur**.